More on Self and Mutual Inductances

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We have seen that An electric current can be induced in a coil by flux change produced by another coil in its vicinity or flux change produced by the same coil. These two situations are described separately in the next two sub-sections. However, in both the cases, the flux through a coil is proportional to the current. ($\phi \propto I$). This implies, in simple conditions that:

$$\frac{\Delta\phi}{\Delta t} \propto \frac{\Delta I}{\Delta t}$$

If we have N coils of wire, it implies that there is N fluxes passing through the conductor, thus, our equation becomes:

$$\mathrm{N}\Delta\phi \propto \Delta\mathrm{I}$$

The constant of proportionality for the above relationship is called inductance. Thus,

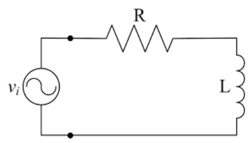
$$\mathrm{N}\Delta\phi = \mathrm{L}\Delta\mathrm{I} \text{ or } \mathrm{N}\Delta\phi = \mathrm{M}\Delta\mathrm{I}$$

As we have seen with resistance and capacitance, inductance depends solely on the intrinsic properties of the material and geometry of the coil(discussed during self-inductance of a solenoid). From Faraday's law we had:

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t} = -\mathrm{L}\frac{\Delta\mathrm{I}}{\Delta t}$$

RL Circuits and Dissipated Energy

RL circuit is a circuit that has resistance and inductance. We have seen RC circuits in the past while discussing capacitors. In this case, we will be seeing RL circuits and our focus will be the impact of inductance in a circuit. Here is a schematic example of an RL circuit:



We can see that EMF of the source is divided between the inductor and the resistor since they are connected in series. Thus, we have:

$$\varepsilon = \mathrm{IR} + \mathrm{L}\frac{\Delta\mathrm{I}}{\Delta\mathrm{t}}$$

When solving for this equation(this is a differential equation that is similar to the one we had in RC circuits) as we did with RC circuits, we will have the following:

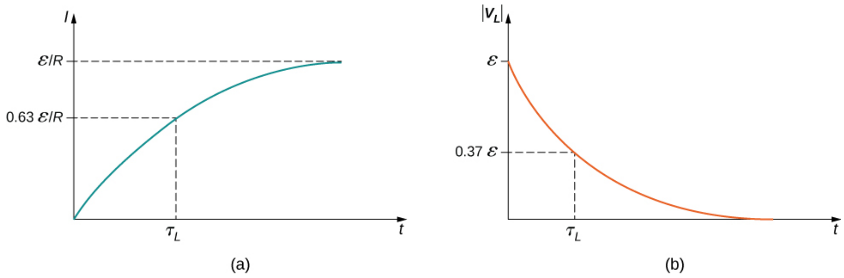
$$I(t) = \frac{\varepsilon}{R}(1 - e^{\frac{-Rt}{L}})$$

We define the quantity inductive(characteristic) time constant as:

$$\tau_L = \frac{L}{R}\text{, thus our equation becomes}$$

$$I(t) = \frac{\varepsilon}{R}(1 - e^{\frac{-Rt}{L}})$$

Let's consider how the current and EMF would behave in such a case.



In **(a)**, we see the how current behaves in the circuit. At the start, there is no current, but as time goes on, the current increases asymptotically to reach $\dfrac{\varepsilon}{R}$ at t. We see that the current will have 63% of its maximum value at the inductive time constant.

$$V_L(t) = -L\frac{\Delta I(t)}{\Delta t}$$
$$V_L(t) = -\varepsilon e^{\frac{-t}{\tau_L}}$$

In case of the induced voltage as a result of inductance(V_L), we have seen that it is directly proportional to the time rate of change of current. That means, it will have its maximum value immediately after the switch is turned on in the circuit and its value decreases along with time. When the current is maximum($\frac{\varepsilon}{R}$), the induced voltage becomes 0. The inductive time constant tells us how fast the voltage decays and also how fast the current exponentially increases. It is an indicative measure of rapid changes in the circuit. The energy stored in an RL circuit is given by the following:

$$E = \frac{1}{2}LI^2$$

Example Problem: What is the characteristic time constant for a 9.00 mH inductor in series with a **3.00 Ω** resistor? (b) Find the induced EMF 8.00 ms after the switch is turned on if the source emf is 10.0V.

a

$$\tau_L = \frac{L}{R} = \frac{9.00 \times 10^{-3}H}{3\Omega} = 3.00 \times 10^{-3}s = 3.00ms$$

b

$$I(t) = (10.0V)e^{-(8.00ms)/3.00ms}$$
$$I(t) = 10.0V \times e^{\frac{-8}{3}}$$
$$I(t) = 10.0V \times 0.0695$$
$$I(t) = 0.695V$$