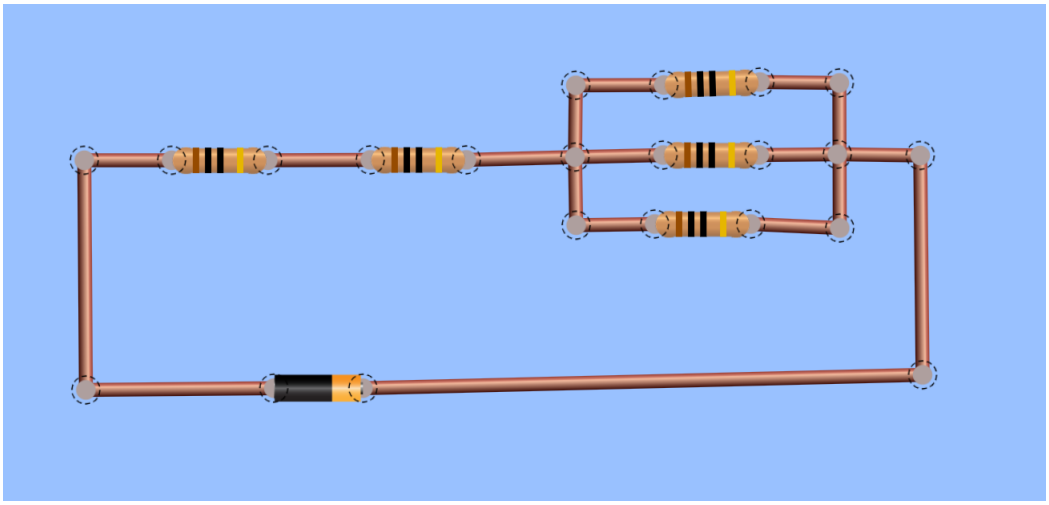
Grade 10 Bonus Quiz Solutions

March, 2022

1. In the circuit given below, there passes through a current of 3A and the voltage supplied by the battery is 7V. Calculate the current and potential difference through each resistor if the resistors have the same resistance.



Solution:

The first thing we need to do is find the effective resistance of the circuit. We have been given the current through the circuit and the voltage provided by the battery. Thus, the effective resistance in the circuit is given by Ohm's Law which is $R_{eff} = \frac{V}{I}$. That means

$$R_{eff} = \frac{7V}{3A} = \frac{7}{3}\Omega$$

Since we have equal-resistance resistors, we can all assume their resistance to be R. Then we can find the effective resistance of the circuit in terms of R. We see that two of our resistors are in series with the battery while the other three are in parallel. To find the effective resistance, we first need to find the effective resistance of the circuits in parallel. If we name the first two resistors R_1 and R_2, we can name the other three resistors R_3, R_4, and R_5. The effective resistance of the three resistors in parallel, R_{345} is given by

$$\frac{1}{R_{345}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$$

Since $R_3 = R_4 = R_5 = R$,

$$\frac{1}{R_{345}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R}$$
$$\frac{1}{R_{345}} = \frac{3}{R}$$
$$R_{345} = \frac{R}{3}$$

The next step is then to find the effective resistance of the circuit. Since R_{345} is in series to R_1 and R_2, we can find the effective resistance of the circuit by simply adding all three as follows

$$R_{eff} = R_1 + R_2 + R_{345} = R + R + \frac{R}{3}$$
$$R_{eff} = \frac{7}{3}R$$

We found the effective resistance early on to be $\frac{7}{3}\Omega$ and now we found the effective resistance in terms of R to be $\frac{7}{3}R$, since these two are equal,

$$\frac{7}{3}\Omega = \frac{7}{3}R$$

Then, we find $R = 1\Omega$. This means that the resistance of each resistor is 1Ω each. Since we have now found the resistance of each resistor, we can find the current and potential difference through each resistor.

Through the first and the second resistors, the current that flows is the same as the current through the circuit because they are connected in series with the battery. That means,

$$I_1 = I_2 = 3A$$

We can then find the voltage through both resistors since we now know both the current and the resistance.

$$V_1 = I_1R_1 = (3A)(1\Omega) = 3V, \text{ and}$$
$$V_2 = I_2R_2 = (3A)(1\Omega) = 3V$$

From our circuit structure, we know that $V_1 + V_2 + V_{345} = 7V$ since the sum of the voltage of resistors connected in series should equal the voltage provided by the battery. Thus

$$V_{345} = 7V - V_1 - V_2 = 7V - 3V - 3V = 1V$$
$$V_{345} = 1V$$

Since R_3, R_4, and R_5 are all connected in parallel, the potential difference along each resistor is the same. So $V_3 = V_4 = V_5 = V_{345} = 1V$. Since we know the potential difference along each resistor and the resistance, we can find the current through them. That means,

$$I_3 = \frac{V_3}{R_3} = \frac{1V}{1\Omega} = 1A$$

$$I_4 = \frac{V_4}{R_4} = \frac{1V}{1\Omega} = 1A$$
$$I_5 = \frac{V_5}{R_5} = \frac{1V}{1\Omega} = 1A$$

Now, we have found the current and potential difference along each resistor and it is:
$I_1 = 3A$ and $V_1 = 3V$
$I_2 = 3A$ and $V_2 = 3V$
$I_3 = 1A$ and $V_3 = 1V$
$I_4 = 1A$ and $V_4 = 1V$
$I_5 = 1A$ and $V_5 = 1V$

2. There are 64 equal-resistance resistors connected in parallel, if the battery supplies a voltage of 8V and the total current in the circuit is 4 A, find the resistance, current, and potential difference through each resistor.

Solution:

Since all the resistors are connected in parallel, we can add the resistance R_eff in terms of R like this;

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_{64}}$$

But we know that each resistance is

$$\frac{1}{R_{eff}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + ...\frac{1}{R} \text{ added 64 times}$$
$$\frac{1}{R_{eff}} = \frac{64}{R}$$
$$R_{eff} = \frac{R}{64}$$

And we also know the effective resistance of the circuit since we have been given the voltage and the current. Thus,

$$R_{eff} = \frac{V}{I}$$
$$R_{eff} = \frac{8V}{4A}$$
$$R_{eff} = 2\Omega$$

Since we know the effective resistance in terms of R and we have found the effective resistance, we can find R.

$$\frac{R}{64} = 2\Omega$$
$$R = 128\Omega$$

That means, the resistance of each of the 64 resistors is 128Ω Since all the resistors are connected in parallel, they all have the same potential difference across and it is 8V. We also have been told the question that they have the same resistance, and thus we can deduce that they all have the same current passing through them. And that current is calculated as shown below:

$$I = \frac{V}{R}$$

$$I = \frac{8V}{128\Omega}$$

$$I = \frac{1}{16}A$$

Each resistor has a voltage of 8V across it and the current passing through each is $\frac{1}{16}A$